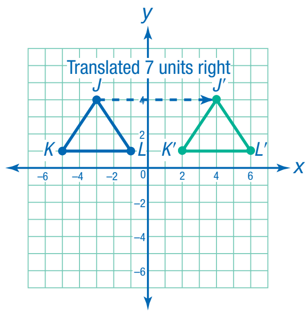
LESSON 2 Translations

UNDERSTAND A **translation** is an operation that slides a geometric figure in the plane. You can think of a translation of a geometric figure as a function in which the input is not a single value, x, but rather a point on the coordinate plane, (x, y). When you apply the function to a point, the output will be the coordinates of the translated image of that point.

You can translate not only individual points but also entire graphs and figures. When you apply a translation function to every point on a figure, the resulting points will form the translated figure. For each **line segment** on the original figure, the translated image will contain either a corresponding **parallel line segment** or a **collinear line segment** of equal length.

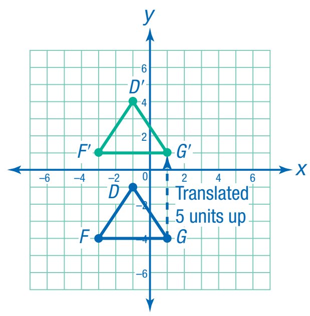
In a **horizontal translation,** the x-coordinate changes, but the y-coordinate stays the same. A horizontal translation of a units can be represented by the function $T(x, y) = (x + a, y)$. If $a > 0$, the figure slides to the right. If $a < 0$, the figure slides to the left.

The transformation shown on the right is the result of applying the function $T(x, y) = (x + 7, y)$ to $\triangle JKL$. In this example, a is a positive number, 7, so the figure slides to the right.



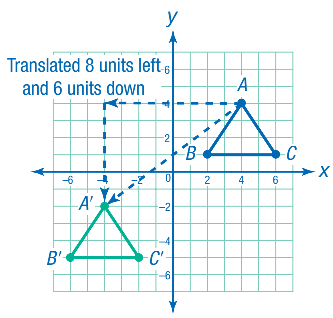
In a **vertical translation**, the y-coordinate changes, but the x-coordinate stays the same. A vertical translation of b units can be represented by the function $T(x, y) = (x, y + b)$. If $b > 0$, the figure slides up. If $b < 0$, the figure slides down.

The transformation shown on the right is the result of applying the function $T(x, y) = (x, y + 5)$ to $\triangle DFG$. In this example, b is a positive number, 5, so the figure slides up.



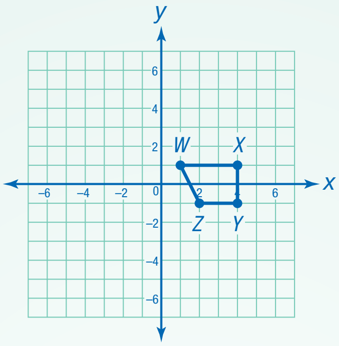
In a slant translation, both the x- and y-coordinates change. Slant translations can be described by the function $T(x, y) = (x + a, y + b)$.

The transformation shown on the right is the result of applying the function $T(x, y) = (x - 8, y - 6)$ to $\triangle ABC$. In this example, a and b are both negative, so the figure slides to the left and down.

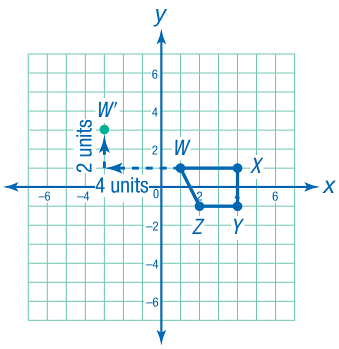


Connect

Translate trapezoid *WXYZ* 4 units to the left and 2 units up to form trapezoid $W'X'Y'Z'$. Identify the coordinates of the vertices of the translated image.

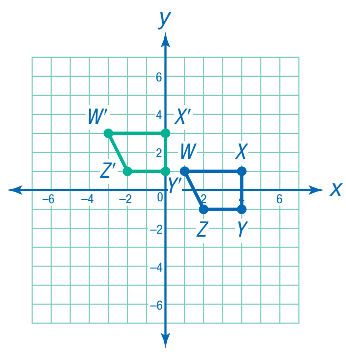


1 Starting at point *W*, count 4 units to the left and 2 units up.



Plot point W' there. Notice that its coordinates are $(-3, 3)$.

2 Translate every other point in the same way—by sliding it 4 units to the left and 2 units up. Then connect the points to form trapezoid $W'X'Y'Z'$.



▶ The coordinates of the vertices of the translated image, $W'X'Y'Z'$, are $W'(-3, 3)$, $X'(0, 3)$, $Y'(0, 1)$, and $Z'(-2, 1)$.

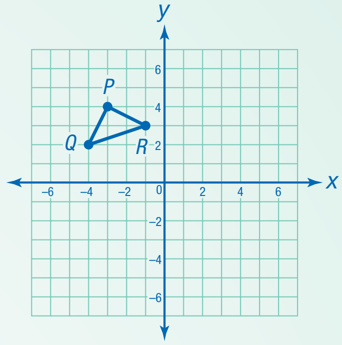
DISCUSS

What function represents the translation that you performed? How do you know?

$T(x, y) =$ ____________________

EXAMPLE A Translate $\triangle PQR$ according to the rule below:

$T(x, y) = (x + 6, y - 1)$



1 Identify the coordinates of the vertices of $\triangle PQR$.

The vertices are $P(-3, 4)$, $Q(-4, 2)$, and $R(-1, 3)$.

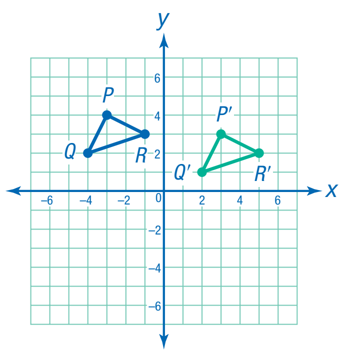
2 Treat each point as an input and substitute it into the rule above to find the coordinates of the translated image.

$T(-3, 4) = (-3 + 6, 4 - 1) = (3, 3)$

$T(-4, 2) = (-4 + 6, 2 - 1) = (2, 1)$

$T(-1, 3) = (-1 + 6, 3 - 1) = (5, 2)$

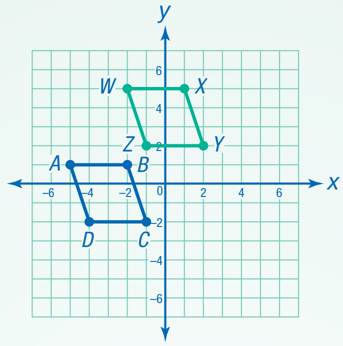
3 Plot points P', Q', and R'. Connect them to form the translated image.



DISCUSS

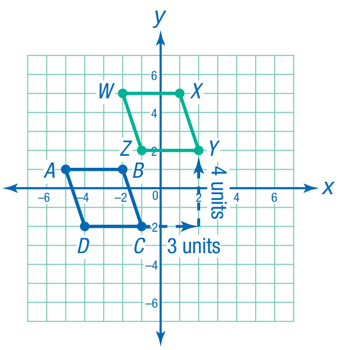
On the diagram in Step 3, trace the path of each vertex of $\triangle PQR$ to its translated image on $\triangle P'Q'R'$. Compare how each point moves from the preimage (or original figure) to the image. Explain what this means about the relationship between the sides of the preimage and the sides of the image.

EXAMPLE B Use a function to describe how parallelogram *ABCD* could be translated so it covers parallelogram *WXYZ* exactly.



1

Describe the slide needed to move vertex *C* of parallelogram *ABCD* onto point *Y*, the corresponding point on parallelogram *WXYZ*.



The diagram shows that point *C* must slide 3 units to the right and 4 units up to move onto point *Y*. Every other point in *ABCD* must slide in the same way.

2

Use a function to describe the translation.

A horizontal translation of 3 units to the right is in the positive direction. It can be represented by the expression $x + 3$.

A vertical translation of 4 units up is also in the positive direction. It can be represented by the expression $y + 4$.

▶ The rule for the translation is:
$T(x, y) = (x + 3, y + 4)$.

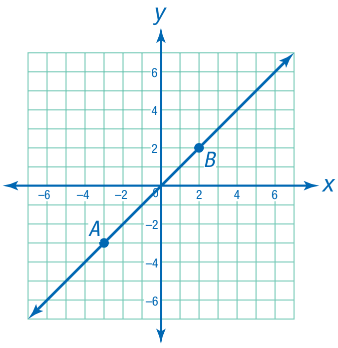
CHECK

Substitute the coordinates of the vertices of parallelogram *ABCD* into the rule $T(x, y) = (x + 3, y + 4)$. Check that the resulting coordinates match those of the vertices of parallelogram *WXYZ*.

Practice

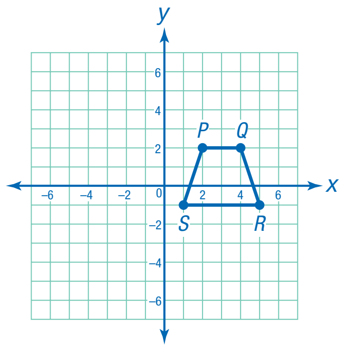
Draw the image for each translation of the given preimage. Use prime (′) symbols to name points on each image.

1. Translate $\overleftrightarrow{AB}$ 3 units to the right.

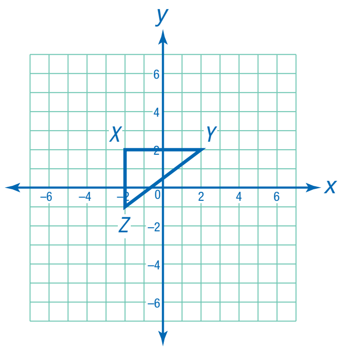


HINT A translation to the right affects the *x*-coordinate.

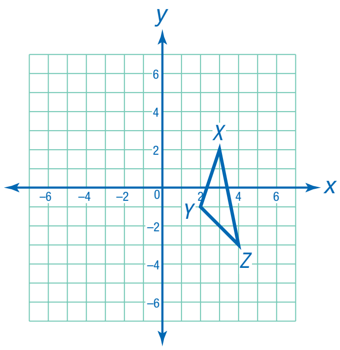
2. Translate trapezoid *PQRS* 7 units to the left and 4 units down.



3. $T(x, y) = (x, y - 4)$



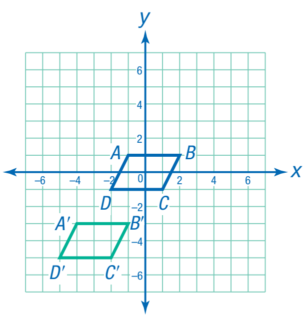
4. $T(x, y) = (x - 8, y + 3)$



REMEMBER The preimage and the image should be the same size and same shape.

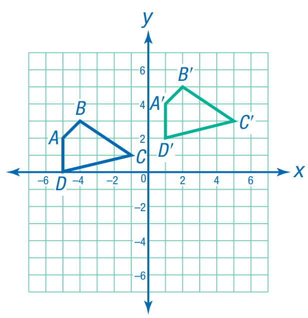
Write a function to describe how the quadrilateral *ABCD* was translated to form *A′B′C′D′* in each graph.

5.



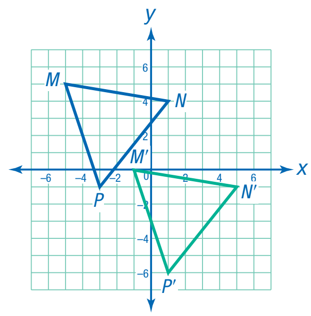
$T(x, y) =$ ______________________

6.



$T(x, y) =$ ______________________

Use the graph on the right for questions 7–9.



7. Name the line segment that is parallel to $\overline{MN}$. ____________

8. Name a line segment that is parallel to $\overline{MP}$. ____________

9. How does $\overline{NP}$ compare to $\overline{N'P'}$?

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Solve.

10. A triangle with vertices $A(1, -3)$, $B(-7, 12)$, and $C(5, 0)$ is translated according to the rule $T(x, y) = (x - 3, y + 9)$. What are the coordinates of the vertices of the translated image?

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11. Point P at $(-4, 3)$ is translated to form its image, point P', at $(6, 1)$. Write a function to represent the translation. If point $R(-5, -6)$ and point $S(1, 2)$ are also translated using that rule, what will be the coordinates of their images?

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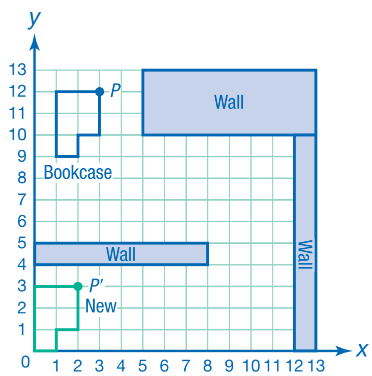
12. **DESCRIBE** A librarian wants to move the bookcase shown in the diagram from its current location to the "New" location. Describe a series of translations that could be used to move the bookcase to its new location, keeping in mind that it cannot be moved through a wall.



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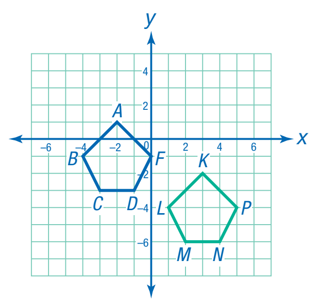
13. **EXPLAIN** Use a function to describe the translation that moves pentagon $ABCDF$ so it covers pentagon $KLMNP$ exactly. Then write the function that moves pentagon $KLMNP$ so it covers pentagon $ABCDF$ exactly. Compare the two functions and explain any differences between them.



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